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THE TOTAL ENTHALPY OF A ONE-DIMENSIONAL NOZZLE FLOW WITH VARIOUS GASES

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SUMMARY

Real-gas thermal properties are used in a method for determining the total enthalpy of a one-dimensional nozzle stream as a function of a simple parameter which includes weight-flow rate, throat area, and total pressure. For a total enthalpy range from about 200 to 10,000 Btu/lb and for total pressures from 1 to 100 atmospheres, charts determined by the method are presented for the case of equilibrium flow of air, nitrogen, carbon dioxide, and argon. Curves for frozen air flow are also included.

INTRODUCTION

As an aid in determining the total enthalpy of a one-dimensional wind-tunnel nozzle flow, it is convenient to have a plot of the total enthalpy as a function of a weight-flow parameter which includes the measurable quantities of weight-flow rate, sonic throat area, and total (reservoir) pressure. In reference 1 a chart of total enthalpy versus weight-flow parameter was computed for equilibrium air flow. The validity of the use of such a chart for determining the total enthalpy of a wind-tunnel air stream heated by an electric arc is demonstrated in reference 2. Because of interest in simulating in wind tunnels the expected flight conditions within planetary atmospheres, it is desirable that charts for determining the total enthalpy of a nozzle flow be available for several gases as well as for air. It is the purpose of this report to present such charts for air, nitrogen, carbon dioxide, and argon. Charts for other gases can be prepared by the method described herein.

NOTATION

A   nozzle cross-sectional area, ft^2

a   isentropic speed of sound, ft/sec

F_γ $\sqrt{\gamma \left(\dfrac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}}$

g   gravitational constant, 32.17 ft/sec^2

h   enthalpy (with datum of $h = 0$ for molecular gas at $0°$ R), Btu/lb

M   Mach number

m   molecular weight of mixture, lb/mole

m_0 molecular weight of undissociated gas, lb/mole

p   pressure, atm

R   universal gas constant, $1545 \frac{\text{ft-lb}}{\text{mole }°R}$

T   temperature, $°R$

u   velocity, ft/sec

w   weight-flow rate, lb/sec

Z   molecular weight ratio, $\frac{m_0}{m}$

α total atom mass fraction

γ isentropic exponent

ρ density, slug/ft^3

$(\ )^{*}$ sonic point (taken at throat)

$(\ )_t$ reservoir or total condition

ANALYTICAL METHOD

A formula for the weight-flow rate of a one-dimensional flow from the reservoir to the sonic throat can be derived if the isentropic exponent γ and molecular weight ratio Z are assumed to remain constant. For equilibrium flow it is shown in reference 1 (charts 12 and 13) that γ remains essentially constant, whereas Z may decrease only 2 to 3 percent for values of h_t to about 10,000 Btu/lb. Hence, the temperature and pressure relations for a perfect gas can be used to derive the weight-flow rate for a real gas. The equations used in the derivation are as follows:

$$\frac{w}{A} = \rho u \qquad \text{[mass continuity]} \qquad (1)$$

$$\rho = \frac{p m_0}{Z R T} \qquad \text{[thermal state]} \qquad (2)$$

$$\frac{T_t}{T} = 1 + \frac{\gamma - 1}{2} M^2 \qquad \text{[adiabatic, perfect]} \qquad (3)$$

2

$$\frac{p_t}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \qquad \text{[isentropic, perfect]} \tag{4}$$

$$M = \frac{u}{a} = \frac{u}{\sqrt{\gamma Z \frac{R}{m_0} T}} \qquad \text{[Mach number]} \tag{5}$$

By combining equations (2) through (5) and substituting into equation (1), we obtain

$$\frac{w}{p_t A} = \sqrt{\frac{\gamma m_0}{ZRT_t}} \, M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)} \tag{6}$$

At the nozzle throat, $M = 1$, $A = A^*$, and

$$\frac{w}{p_t A^*} = \sqrt{\frac{\gamma m_0}{ZRT_t} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}} \tag{7}$$

For w in pounds per second, p_t in atmospheres, and A^* in square feet,

$$\frac{w}{p_t A^*} = 2117 \sqrt{\frac{g \gamma m_0}{ZRT_t} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}} \qquad \frac{\text{lb/sec}}{\text{atm ft}^2} \tag{8}$$

where T_t is in $^\circ R$, $g = 32.17$ ft/sec^2, and $R = 1545$ ft-lb/mole $^\circ R$.

For convenience of use with various specific gases equation (8) can be rewritten as

$$\frac{w}{p_t A^*} = \frac{CF\gamma}{\sqrt{ZT_t}} \qquad \frac{\text{lb/sec}}{\text{atm ft}^2} \tag{9}$$

where

$$F\gamma = \sqrt{\gamma \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}} \qquad \text{and} \quad C = 2117 \sqrt{\frac{g m_0}{R}}$$

Values of C (consistent with the units used above) for various gases with molecular weights, m_0, are given in the following table:

Gas	m_O	C
CO_2	44.01	2028
A	39.94	1930
O_2	32.00	1727
Air	28.97	1645
N_2	28.02	1616
He	4.00	611
H	2.02	434

Because it may be desirable to test in mixtures of nitrogen and carbon dioxide for planetary atmosphere simulation, values of m_O and C for various $N_2 - CO_2$ mixtures are also given as follows:

Percent N_2	Percent CO_2	m_O	C
100	0	28.02	1616
95	5	28.82	1640
90	10	29.62	1662
85	15	30.42	1685
80	20	31.22	1707
70	30	32.82	1750
60	40	34.42	1792
50	50	36.01	1833
40	60	37.61	1873
30	70	39.21	1913
20	80	40.81	1951
10	90	42.41	1989
0	100	44.01	2028

For specific values of p_t and h_t, corresponding values of T_t, Z, and γ can be determined from gas tables and/or charts, and then $w/p_t A^*$ can be readily computed from equation (9).

To check the accuracy of the present method the variation of $w/p_t A^*$ with h_t for equilibrium air determined by this method was compared with that determined by the more exact method of reference 1. In reference 1 w/A^* was determined by plotting free-stream density ρ times velocity u versus static pressure p along the nozzle axis for specific values of p_t and h_t. Then w/A^* was taken as the maximum value of ρu. Because this method requires a point by point nozzle-flow calculation and a plot of ρu versus p for each

specified h_t and p_t, it is more tedious and time consuming than the present method, but inherently it can be expected to be more exact. However, over the total enthalpy and total pressure ranges considered (see fig. 1), the agreement between the methods is excellent.

RESULTS AND DISCUSSION

Charts for Various Gases in Equilibrium Flow

For equilibrium flow from the reservoir to the sonic throat the method outlined in the previous section was used to compute the variation of $w/p_t A^*$ with h_t for air, nitrogen, carbon dioxide, and argon. Values of p_t of 1, 10, and 100 atmospheres were assumed. The resulting charts are presented in figures 2 through 5. As shown on the charts the range of the total enthalpies considered was from about 200 to 10,000 Btu/lb. Symbols on the charts indicate computed points.

For the calculations gas thermal properties from several sources were used. The National Bureau of Standards' tables of reference 3 were used for all of the gases for values of h_t from the lowest plotted in figures 2 through 5 to the highest given in the NBS tables. (The highest values of h_t from the NBS tables are indicated on the figures.) For values of h_t higher than those given in the NBS tables, properties from references 4, 5, 6, and 7 were used for air, nitrogen, carbon dioxide, and argon, respectively.

During the course of the calculations it was noted that the function F_γ generally changed only a small amount for the ranges of p_t and h_t considered. For example, on the plot of F_γ versus γ in figure 6 the limits in the variation of γ (and hence F_γ) for air are reasonably close for the h_t range of interest in current arc-heated wind-tunnel facilities (h_t from about 1,000 to 10,000 Btu/lb at p_t from 1 to 100 atm). Over this h_t range approximate values of F_γ could be used for each gas and would result in maximum errors in $w/p_t A^*$ of about ±3 percent. The approximate values of F_γ are as follows:

Gas	F_γ
Air	0.65
N_2	.66
CO_2	.63
A	.69

If the gas deviates from equilibrium or becomes frozen between the reservoir and the nozzle throat, the equilibrium-flow charts will give erroneous values of h_t for measured values of w/p_tA^*. To estimate the maximum limits of uncertainty in h_t as a result of nonequilibrium air flow, additional calculations of h_t versus w/p_tA^* have been made for frozen flow with complete vibrational excitation and with no vibrational excitation. It should be noted that these calculations were made for the assumption of isentropic air flow downstream of the reservoir. In figure 7 the results are compared with those for equilibrium flow. The flows were assumed frozen at the reservoir $(M = 0)$ condition. Equation (9) was used to compute the curves for these frozen flows as well as for the equilibrium flow. In the calculations only the values of F_γ change for the different flows, since F_γ is a direct function of γ (fig. 6) which is, in turn, a function of Z as shown in figure 8. The comparisons in figure 7 clearly demonstrate that for high values of h_t (low w/p_tA^*) considerable uncertainty in determining the correct h_t for a wind tunnel can result if the flow from the reservoir to the throat is not close to equilibrium.

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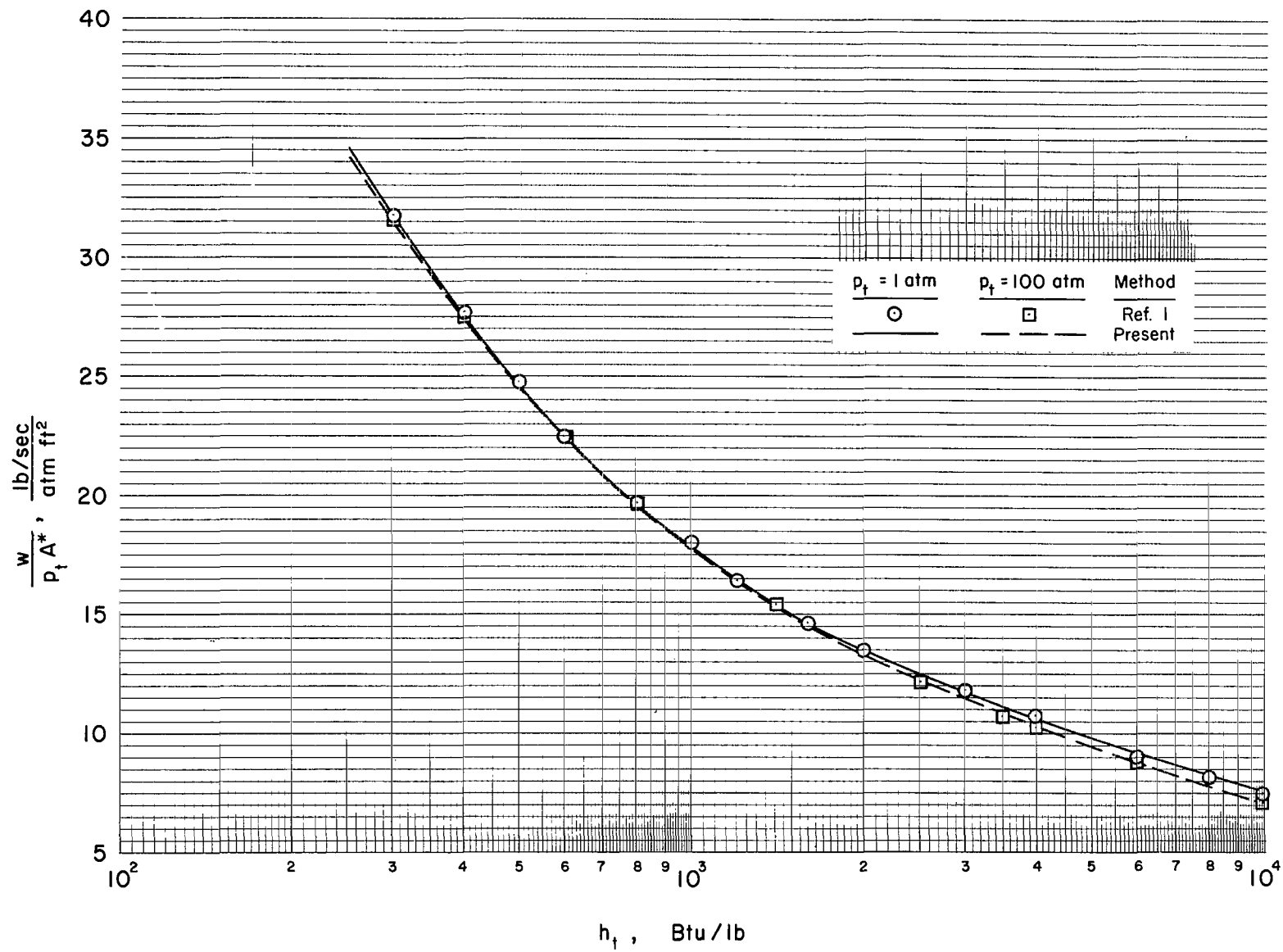
Figure 1.- Comparison of the present method with that of reference 1 for equilibrium air.
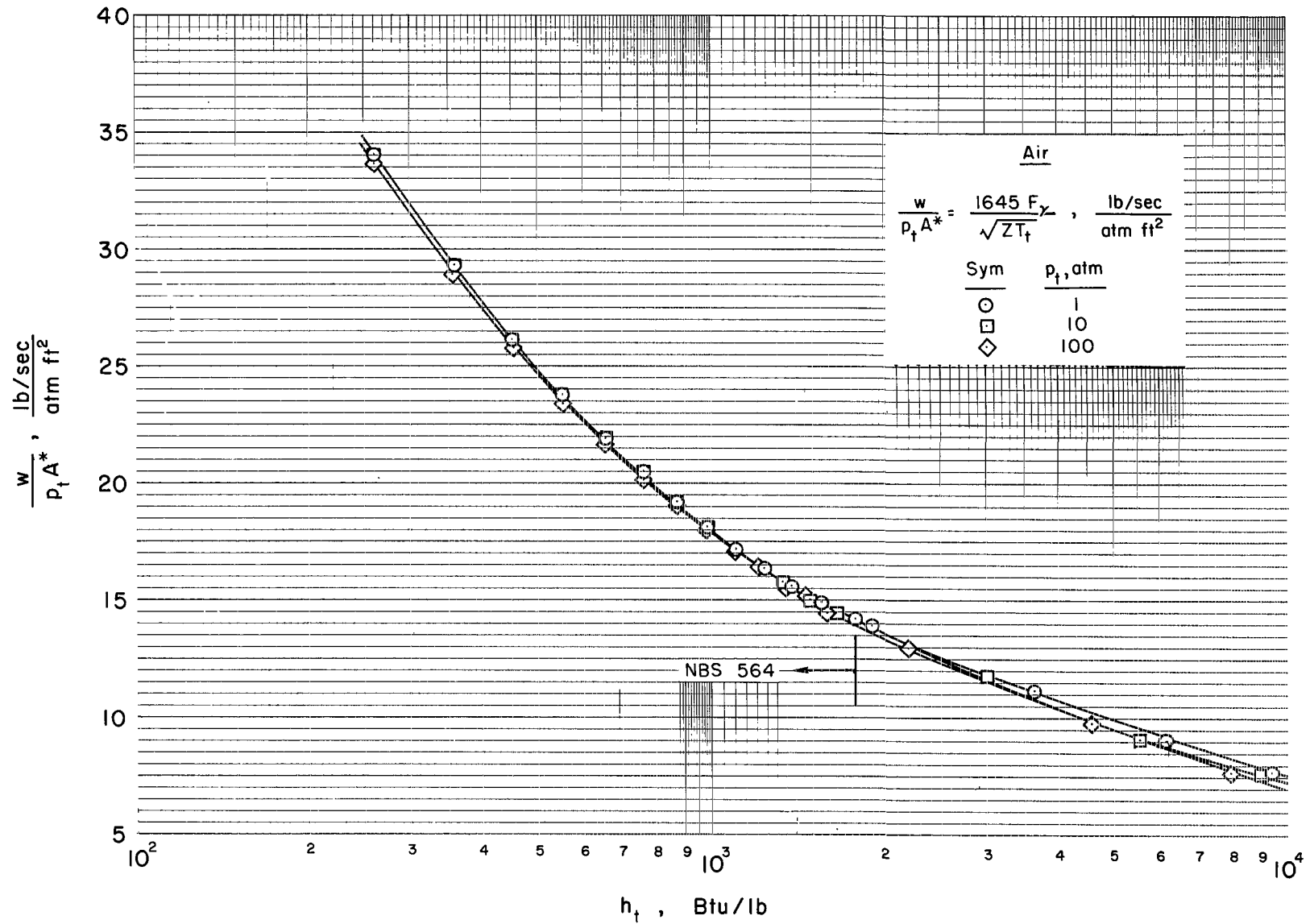
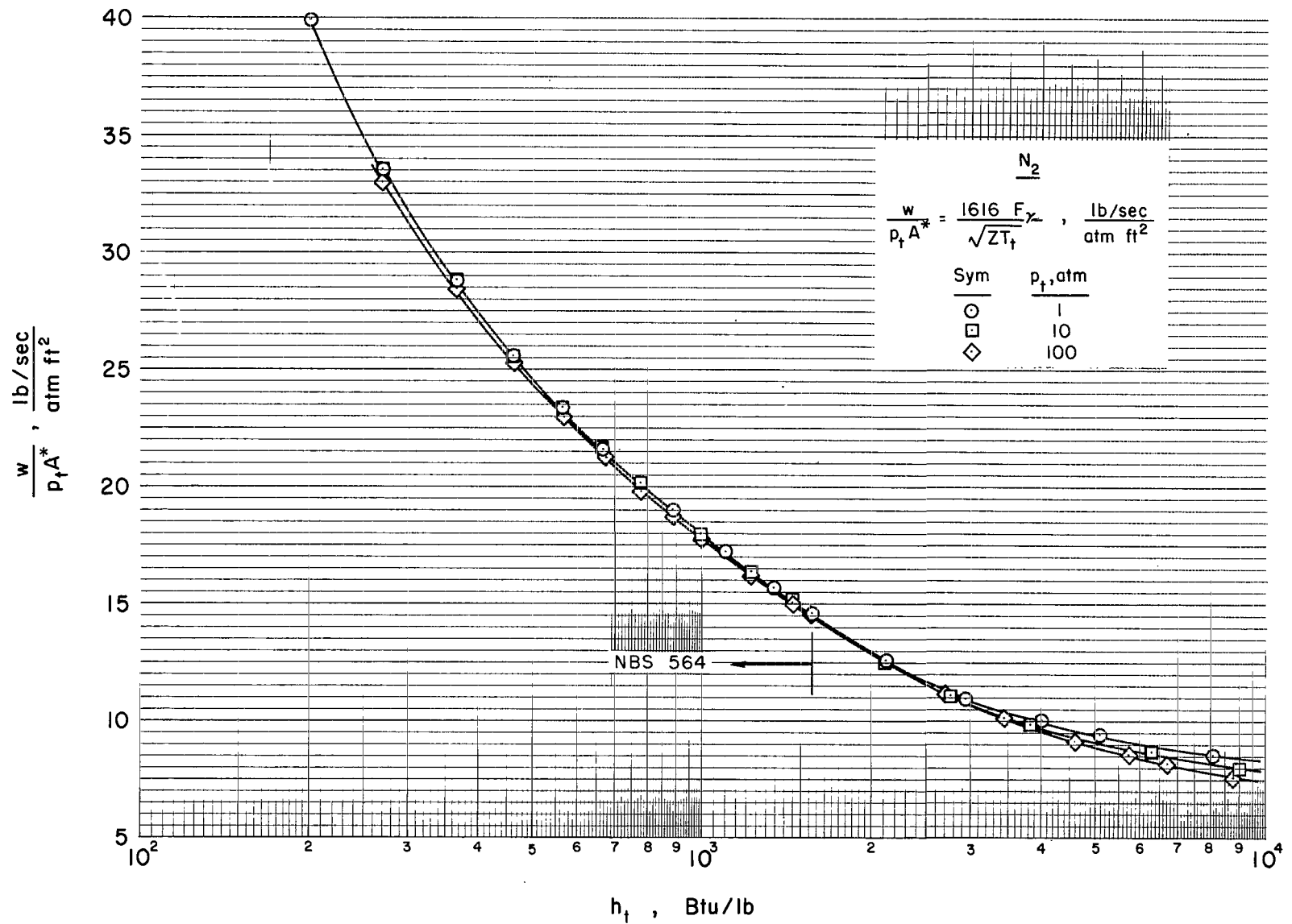Figure 2.- Variation of weight-flow parameter with total enthalpy for equilibrium air.

Figure 3.- Variation of weight-flow parameter with total enthalpy for equilibrium nitrogen.
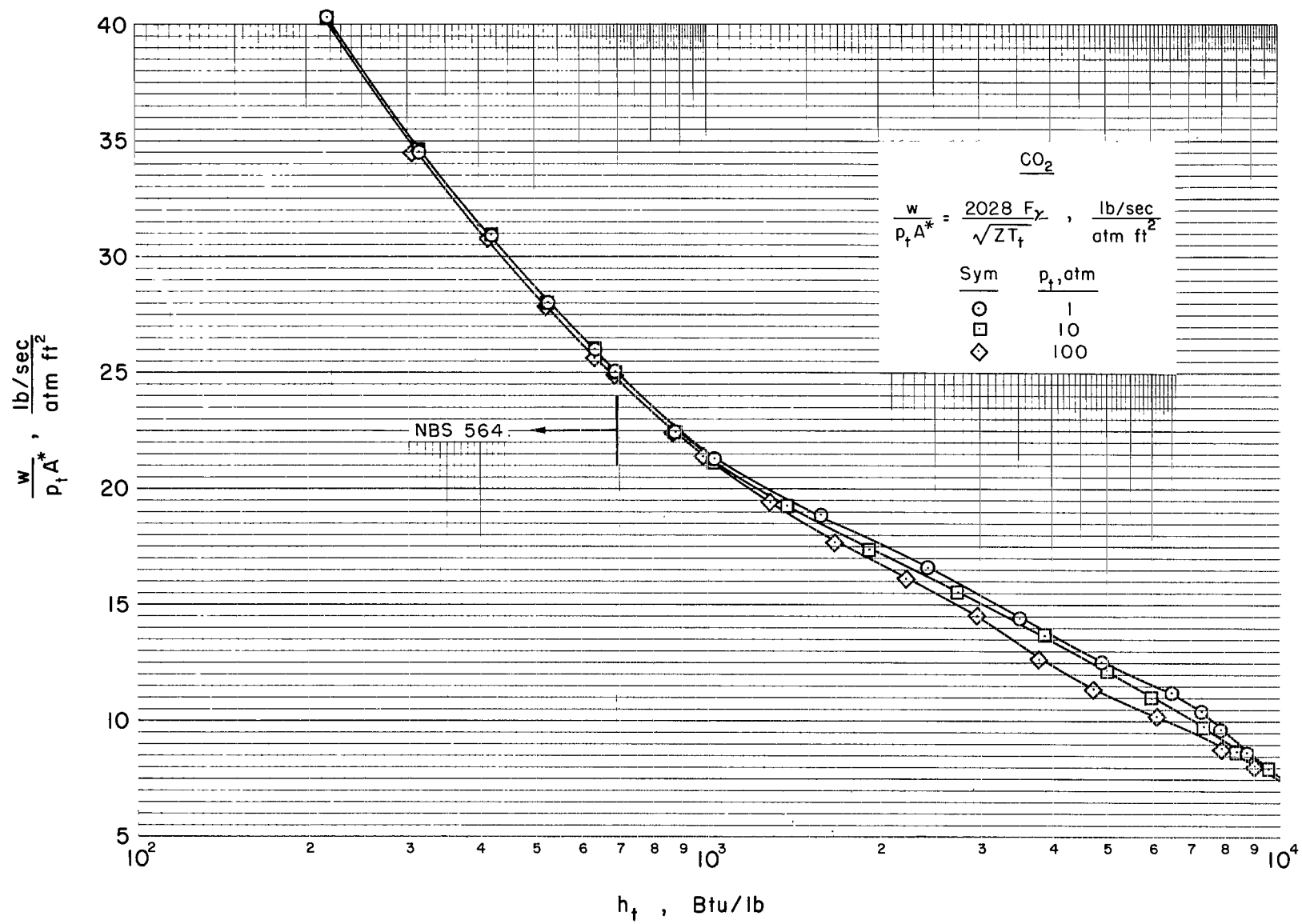
Figure 4.- Variation of weight-flow parameter with total enthalpy for equilibrium carbon dioxide.
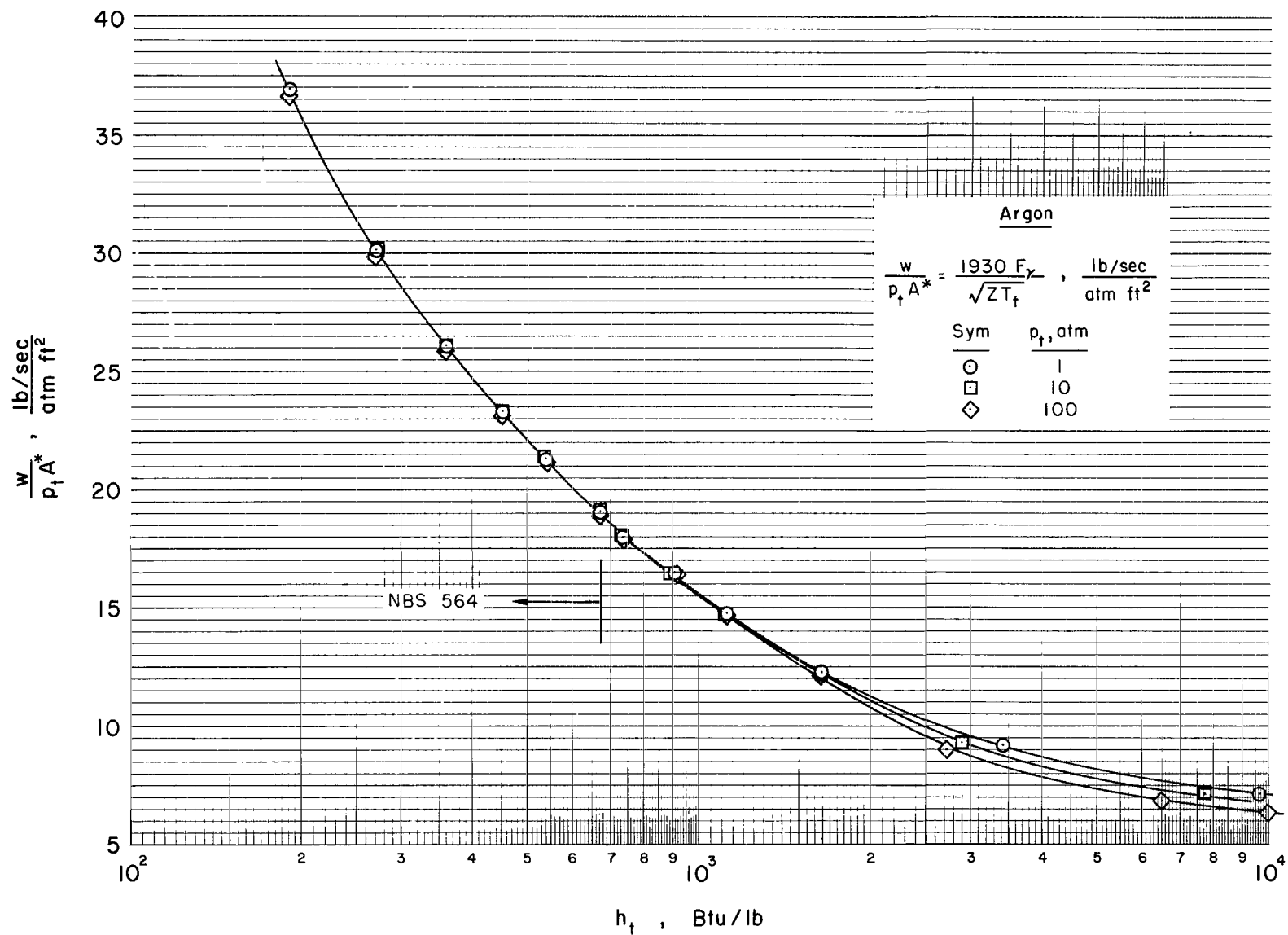
Figure 5.- Variation of weight-flow parameter with total enthalpy for equilibrium argon.
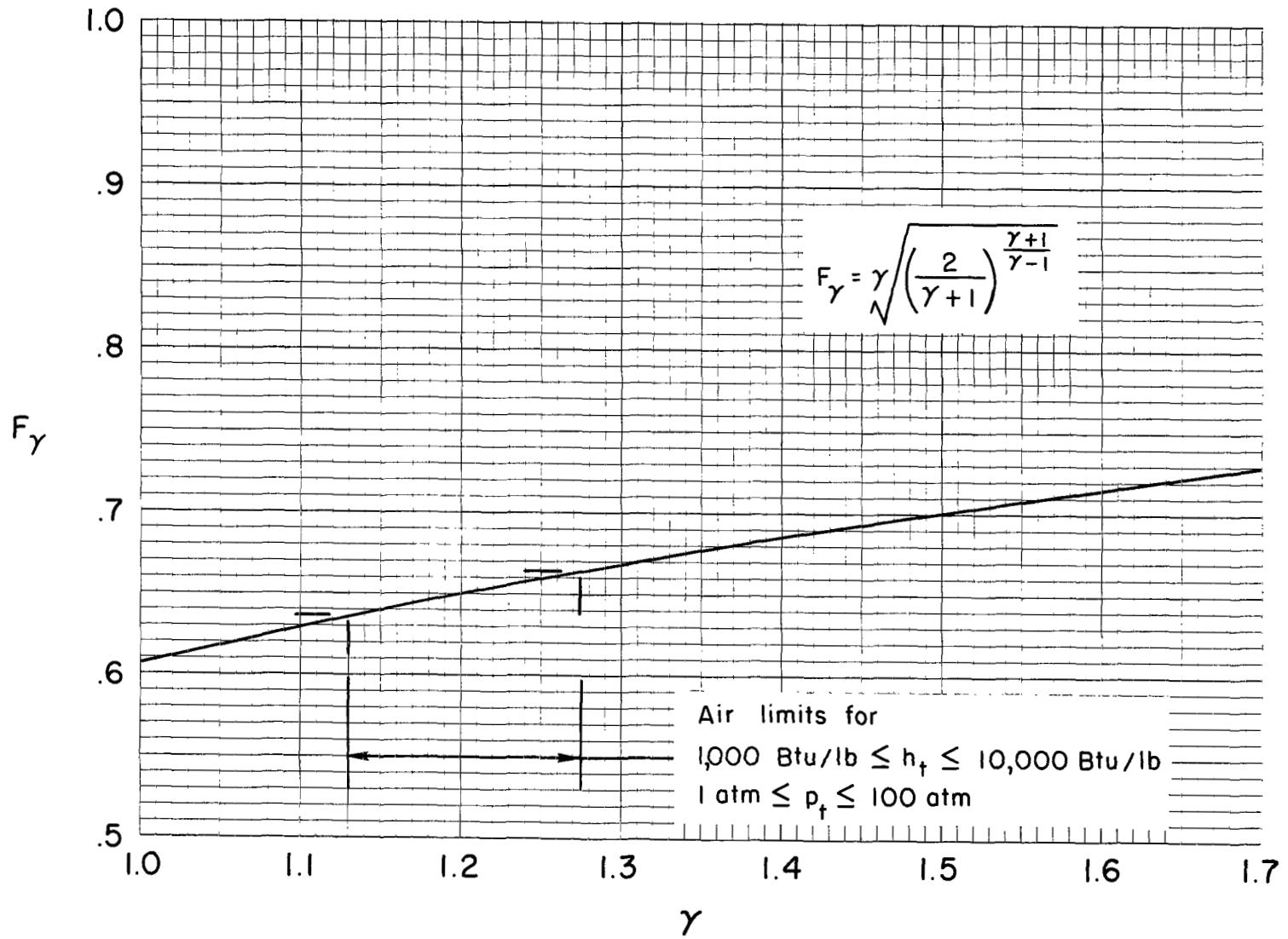
The chart shows the weight-flow parameter $\frac{w}{p_t A^*}$ in units of $\frac{lb/sec}{atm\ ft^2}$ plotted against h_t, Btu/lb.

Legend:

Argon

$$\frac{w}{p_t A^*} = \frac{1930\ F_\chi}{\sqrt{Z T_t}}\ ,\ \frac{lb/sec}{atm\ ft^2}$$

Sym	p_t, atm
⊙	1
⊡	10
◇	100

NBS 564

$$F_\gamma = \gamma \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

Air limits for
$1,000 \text{ Btu/lb} \leq h_t \leq 10,000 \text{ Btu/lb}$
$1 \text{ atm} \leq p_t \leq 100 \text{ atm}$
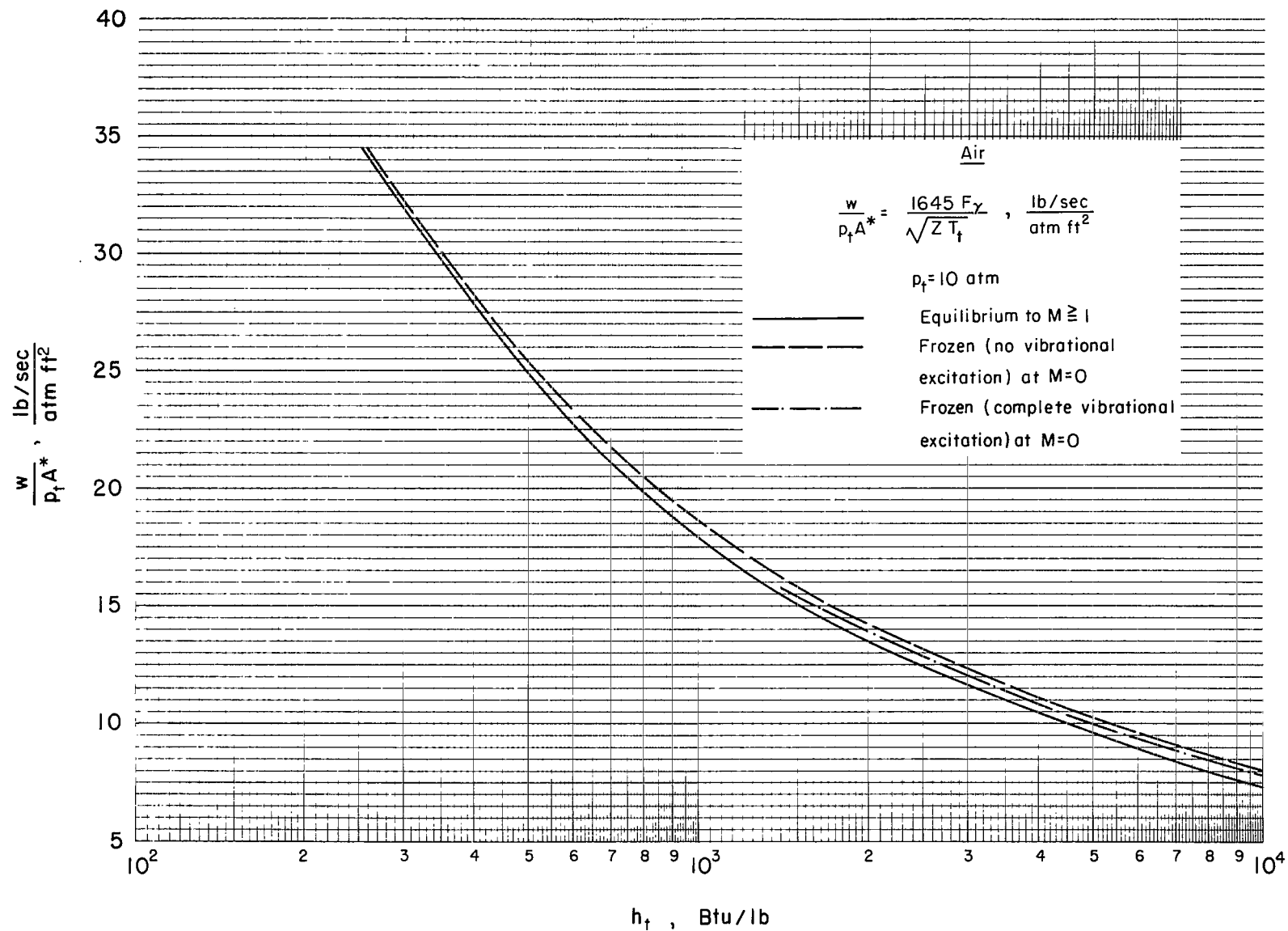
Figure 6.- Variation of F_γ with γ.

Figure 7.- Comparison of the variation of weight-flow parameter with total enthalpy for equilibrium and frozen air flow.
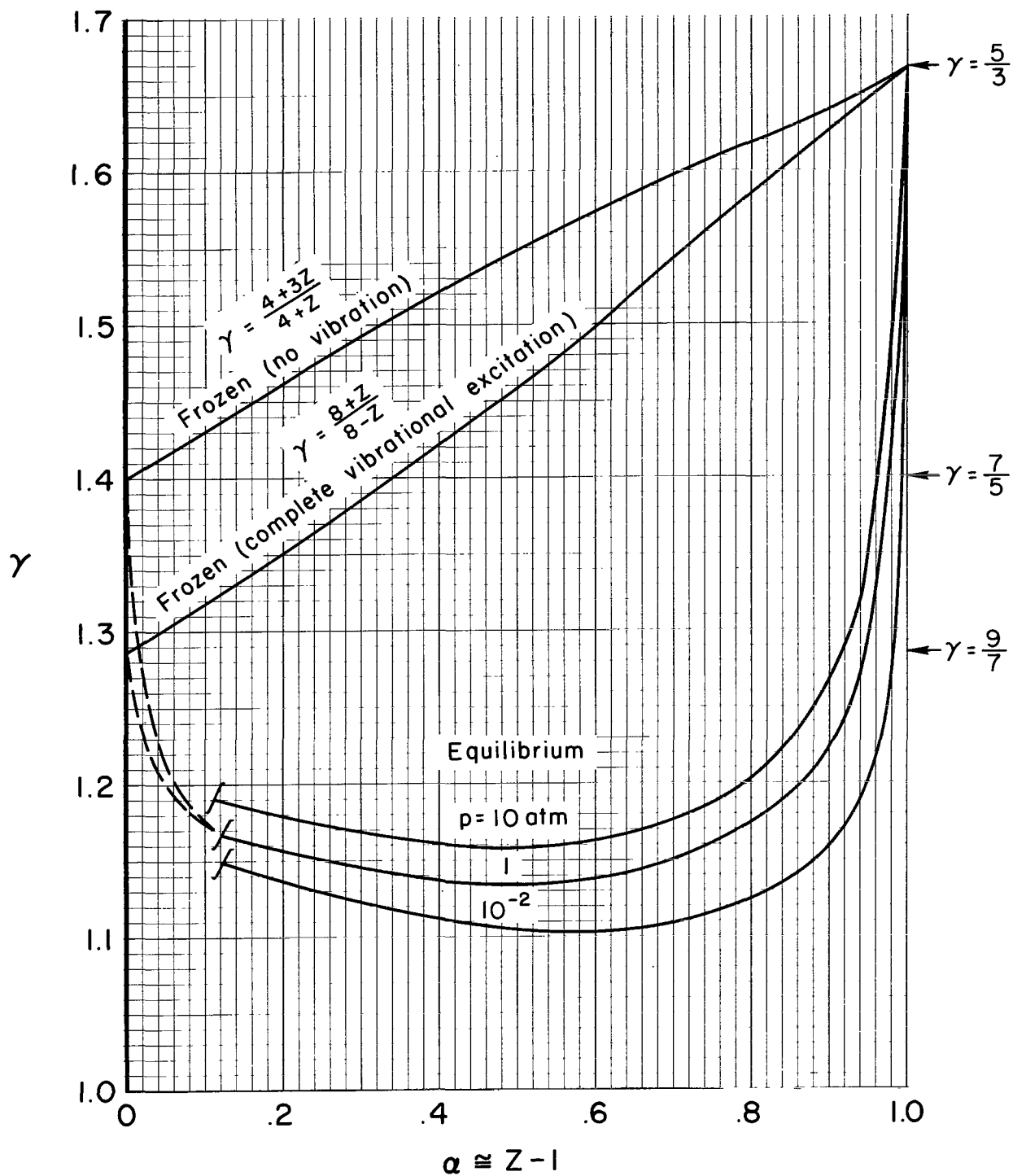
Figure 8.- Variation of γ with $z - 1$ for frozen and equilibrium air (from ref. 8).